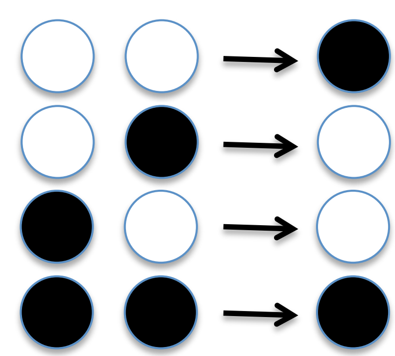
Silly game (0)

An arbitrary positive number of black and white balls is put into an urn. The player repeats the following moves: he takes two balls at random from the urn; if those two balls have the same color he throws one black ball into the urn, otherwise he returns one white ball into the urn.

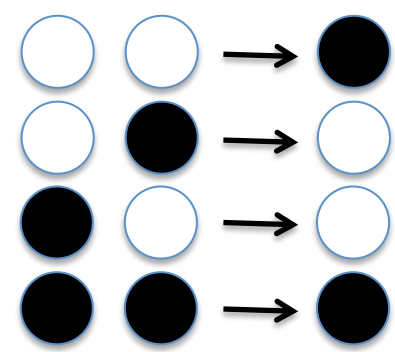
Because each move decreases the total number of balls into the urn by 1, the game is guaranteed to terminate after a finite number of moves with exactly 1 ball in the urn.

What can we say about the color of the final ball when we are given the initial contents of the urn?



Silly game - Solution (0)

The actions of the game keep the parity of the white balls as an invariant, i.e. the parity of the white balls doesn't change. Therefore, if we start with an odd number of white balls then the last ball will be white, but if we start with an even number of white balls the last ball will be black.



Silly game (1)

"Of two known integers between 2 and 99 (bounds included) a person P is told the product and person S is told the sum. When asked whether they know the two numbers, the following dialog takes place:

- P: "I don't know them."
- S: "I knew that already."
- P: "Then I know the two numbers."
- S: "Then I now know them too."

With the above data we are requested to determine the two numbers and to establish that our solution is unique."

[0] E. W. Dijkstra, EWD 666, "A Problem Solved in My Head" (undated)

Silly game - Solution (1)

Reasoning:

1: The product is not the product of two primes.

2: The product contains no prime factor >= 53.

3: The numbers are such that points (1) and (2) can be deduced from the sum.

4: Point (1) excludes for the sum all values that can be written as the sum of two primes (including all even numbers).

5: The first set of possible values for the sum are
S0={11,17,23,27,29,35,37,41,47,51,53}

Silly game - Solution (1)

6: For P to make the third remark, the product cannot be written in more than one way as x * y such that s + y is a member of S0. It is however easy if the product is (a prime * a power of two), since all elements in S0 are odd.

7: Based on S's final remark, remove from S0 all values that can be written in more than one way as prime * power of 2.

8: Possible sums are now S1={17,29,41,53}

9: By dealing with each one of those possibilities and figuring out what is decidable, the only option is 4 and 13.

Silly game – Solution (1)

The unique solution is 4 and 13.

For a detailed explanation see:
E. W. Dijkstra, EWD 666, "A Problem Solved in My Head"
[www.cs.utexas.edu/~EWD/ewd06xx/EWD666.pdf]

Silly game (2)

"Suppose we play a two-player game played with identical coins on a table. We have a bag of coins with as many as we need. The two players alternatively take a coin from the bag and place it on the table.

The rules of the game forbid the coin to sit on top of another coin on the table, but it could hang off the table as long as it does not fall off. The player who puts the last coin on the table wins the game.

Is there an algorithm for one of the players to win always?" [0]

[0] Sorin Istrail, Storytelling about Lighthouses. Conduit (19), Brown University.

Silly game - Solution (2)

We need to assume that the table is symmetric with respect to the center.

The first player always puts his first coin on the center of the table. The second player puts his coin somewhere on the table that is free, and then the first player makes his move in exactly the symmetric point to the center, which is always free by the invariant. And so on, mutatis mutandis. The invariant assures the correctness of the algorithm no matter how many moves/how big the table. [0]

[0] Sorin Istrail, Storytelling about Lighthouses. Conduit (19), Brown University.