The class $\#P$ is the class of functions that count the number of solutions to problems in $NP$. Since very few counting problems can be exactly computed in polynomial time (e.g., counting spanning trees), the interest of the community has turned to the complexity of approximating them. The class $\#PE$ of problems in $\#P$ with decision version in $P$ is of great significance.

We focus on a subclass of $\#PE$, namely $\text{TotP}$, the class of functions that count the total number of paths of NPTMs. $\text{TotP}$ contains all self-reducible $\#PE$ functions and it is robust, in the sense that it has natural complete problems and it is closed under addition, multiplication and subtraction by one.

We present logical characterizations of $\text{TotP}$ and two other robust subclasses of this class, building upon two seminal works about descriptive complexity for classes of counting problems [1, 2]. Specifically, to capture $\text{TotP}$, we use recursion on functions over second-order variables which, we believe, is of independent interest.

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