

## REPORT ON THE GRAPH ISOMORPHISM PROBLEM

### Dagstuhl Seminar 15511 on the Graph Isomorphism Problem 13–18 December, 2015

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In 1977, Read and Corneil published a paper with the title *The Graph Isomorphism Disease*, in reference to the infectious nature of the problem and the fact that so many who have studied it have become obsessed with it. This last December saw the largest ever gathering of the afflicted, at Schloss Dagstuhl. The occasion was a five-day seminar on *the Graph Isomorphism Problem* organised by the author of this report, along with Laszlo Babai, Pascal Schweitzer and Jacobo Torán, and attended by 44 participants <http://www.dagstuhl.de/en/program/calendar/semhp/?semnr=15511>.

When we first conceived this seminar, in the autumn of 2014, our motivation was that there was a revival of interest in the problem among a number of distinct research communities. Indeed, while the graph isomorphism problem was intensively studied from the point of view of computational complexity in the 1980s and early 1990s, in later years progress became slow and interest in the problem stalled. However, recent years have seen the emergence of a variety of results related to graph isomorphism in a number of research areas including algebra, logic, combinatorial optimization and parameterized algorithms not to mention graph theory itself. At the same time, practical tools for solving graph isomorphism have improved in leaps and bounds and can now apparently solve all practical instances that arise from applications. Thus, the idea of the seminar was to bring together researchers who are working within these diverse communities but who share a common interest in algorithms for the graph isomorphism problem and get them to exchange insights. We felt that the time for a seminar devoted to the graph isomorphism problem had come. We could not have imagined how right we would turn out to be.

A few weeks before the seminar, the research community was electrified by the news that Laci Babai had announced that the graph isomorphism problem is in quasi-polynomial time. This stunning news caught the attention not only of complexity theorists and the wider computer science community but also attracted media attention. It has been described by some as the most significant complexity-theoretic breakthrough since the turn of the century. To put it in context, recall that the graph isomorphism problem is one of the few natural problems that is clearly in NP but not known to be either NP-complete or in P. Its status as a so-called “NP-intermediate” problem has been cemented through structural

complexity results showing that if the problem were NP-complete, this would imply an implausible collapse in the polynomial-time hierarchy. Before Babai's recently announced result, the asymptotically fastest known algorithm for graph isomorphism was one published in 1983 by Babai and Luks, which has a running time of  $2^{O(\sqrt{n \log n})}$ . The recent announcement gives an upper bound on the complexity of the problem of  $2^{O((\log n)^c)}$  for a small constant  $c$ , which marks a dramatic improvement. Thus, the problem is close to being in P and is certainly solvable asymptotically faster than we expect of NP-complete problems.

At the seminar, we scheduled four hours of talks, spread over the first two afternoons for Laci Babai to present the main ideas involved in his proof. It was an intense blackboard presentation given with great enthusiasm. In the event, in response to demand from participants for more details, Laci gave an extra hour-long unscheduled presentation on Wednesday afternoon, after the traditional Dagstuhl excursion. The paper containing the full details of the proof (<http://arxiv.org/abs/1512.03547>) was released on the arXiv on the first day of the seminar, giving participants the opportunity to consult it for details and it generated vivid discussion.

Apart from Laci's presentation, which was the centre piece of the seminar, there were a number of excellent talks including expository talks on recent advances in a variety of aspects of the graph isomorphism problem. Brendan McKay and Adolfo Piperno each gave an hour-long talk about *Practical Aspects of the Graph Isomorphism Problem*. They discussed the algorithms involved in nauty and Traces respectively, the two fastest graph isomorphism packages. It is striking that the algorithms that are asymptotically fastest in theory are very different to the ones that prove to be the fastest in practical implementations. Brendan described them as distinct galaxies. We had a delightful expository presentation by Albert Atserias titled *Graph Indistinguishability through Hierarchies of Relaxations* on approaching graph isomorphism through linear programming relaxations, which includes a surprising connection to logical definability. Jacobo Torán spoke about *Complexity Classes and Graph Isomorphism* reviewing results on structural complexity. He mentioned a number of intriguing and sometimes exotic complexity classes which have been used to establish upper and lower bounds on the graph isomorphism problem. An extensive survey by Pascal Schweitzer on *Parameterizations and the Graph Isomorphism Problem* covered results on the parameterized complexity of graph isomorphism, under a variety of different graph parameters.

Other noteworthy extended talks included a presentation by Martin Grohe on *Decomposition Techniques for Graph Isomorphism Testing* where he described the structural decompositions of graphs that underlie his results on isomorphism for graph classes with excluded minors. There were three separate talks, by James

Wilson on *New Perspectives in Group Isomorphism*, Eugene Luks on *Group Isomorphism with Fixed Composition Series* and David Rosenbaum on *Bidirectional Collision Detection and Group Isomorphism* which between them covered recent progress on the group isomorphism problem. Laci Babai identifies this as the main barrier to further improvements in graph isomorphism algorithms. In addition, the seminar included fourteen other presentations of varying lengths that covered a diverse range of work and demonstrated the vibrant diversity of research around graph isomorphism.

The seminar was decidedly a memorable event. Some participants even called it historic. During the meeting, a number of participants asked me if the timing of the seminar, so close to the announcement of Laci Babai's result, was coincidental. I joked that by deciding to organise the seminar, we had given Laci a deadline to come up with something to talk about. He most certainly delivered.